14th International Conference on Ordered Statistical Data 2022

BOOK OF ABSTRACTS

May 24th - 27th, 2022

Lloyd's Baia Hotel
Via E. de Marinis, 2, 84019, Vietri sul Mare (SA)

FOREWORD

Continuing the series of Conferences in Mysore, India (2000), Warsaw, Poland (2002-04), Izmir, Turkey (2005), Mashad, Iran (2006), Amman, Jordan (2007), Aachen, Germany (2008), Zagazig, Egypt (2010), Murcia, Spain (2012), Bedlewo, Poland (2014), Piraeus, Greece (2016), Cadiz, Spain (2018) the Local Organizing Committee is pleased to welcome you to the 14th International Conference on Ordered Statistical Data which will be held at Lloyd's Baia Hotel in Vietri sul Mare (SA), Italy, May 24th - 27th, 2022.

The conference will bring forth recent advances and trends in the mathematical theory of ordered statistical data, in order to facilitate the exchange of research ideas, promote collaboration among researchers from all over the world, and contribute to the further development of the field.

The meeting will be dedicated to all aspects of ordered statistical data, including:

Approximations
Bounds
Characterisations
Recurrence Relations
Distribution Theory and Probability Models
Stochastic Orders
Reliability Theory and Survival Analysis
Censoring
Concomitants
Statistical Interference
Applications of Ordered Data
Information and Entropies
Nonparametric Methods
Ranked Set Sampling
Asymptotic Theory
Applications in Biology, Social Sciences, Environmental Sciences.

Vietri sul Mare

May 2022

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Monday 23^{rd} **May**

18:00-20:00 Registration

Tuesday 24^{th} **May**

8:00-9:00 Registration

9:00-9:30 Opening Ceremony

9:30-11:00 **Session 1** - *Dependence and copulas*
Chairman: Miguel Angel Sordo

- Felix Belzunce: *Recent advances in the comparison of dependent random variables*
- Florentina Suter: *Some Information Measures for the Concomitants of m-GOS from a Gumbel's Bivariate Exponential Distribution*
- Jorge Navarro: *Predicting record values by using bivariate distortions*

11:00-11:30 Coffee break

11:30-13:00 **Session 2** - *Reliability Theory and Measures of Discrimination*
Chairman: Francesco Buono

- Antonio Di Crescenzo: *Some results on the weighted mean inactivity time function*
- Agnieszka Goroncy: *Bounds on expected order statistics based on the monotone reversed failure rate distributions*
- Francesco Buono: *A unified formulation of entropy and its application*

13:00-14:45 Lunch

14:45-15:45 **Plenary talk**

Narayanaswamy Balakrishnan, McMaster University, Canada
On Linear Prediction
Chairman: Felix Belzunce

15:45-16:15 Coffee break

16:15-18:15 **Session 3** - *Censored data and record values*
Chairman: Erhard Cramer

- Stefan Bedbur: *Confidence bands for exponential distribution functions under progressive type-II censoring*
- Nikolay Nikolov: *Spacings in a step-stress model with censored exponential life-Times*
- Erhard Cramer: *Structural properties of (progressive) hybrid censoring schemes*
- Christina Empacher: *Prediction of Future Sports Records Based on Record Values*

19:30 Cocktail dinner

Wednesday 25^{th} **May**

8:30-9:00 Registration

9:00-11:00 **Session 4** - *Stochastic orders and order statistics*
Chairman: Tomasz Rychlik

- Alfonso Jose Bello Espina: *Relaxing the increasing convex order: po-tail value at risk order*
- Enrico Scalas: *Continuous-time statistics and generalized relaxation equations*
- Miguel Angel Sordo: *Comparing strong risk aversion in Yaari's dual theory of risk*
- Patricia Ortega Jimenez: *Comparisons of VaR and CoVaR in terms of the value of the conditional variable*

11:00-11:30 Coffee break

11:30-13:00 **Session 5** - *Poster Session:*

Francesco Buono; Camilla Calì; Marco Capaldo; Virginia Giorno; Fabian Kachele; Antonio Lepore; Claudio Macci; Enrica Pirozzi; Nuria Torrado

13:00-15:00 Lunch

15:00-16:00 **Plenary talk**

Fabio Spizzichino, Università degli Studi di Roma "La Sapienza", Italy
The method of multivariate conditional hazard rates: recent developments and applications.
Chairman: Maria Longobardi

16:00-16:30 Coffee break

16:30-18:00 **Session 6** - *In honour of Fabio Spizzichino*
Chairs: Franco Pellerey and Maria Longobardi

- Emilio De Santis: *Paradoxical aspects in probability, game theory and voting theory*
- Rachele Foschi: *Reversing Conditional Orderings*
- Giovanna Nappo: *Minimally stable lifetimes: marginal distributions of order statistics by means of m.c.h.r functions and DD copulas*

20:30 Gala dinner

Thursday 26^{th} **May**

8:45-9:45 **Plenary talk**

Alfonso Suárez-Llorens, Universidad de Cádiz, Spain
Characterization of distributions based on shape measures.
Chairman: Antonio Di Crescenzo

9:45-11:15 **Session 7** - *Some results related to the convex transform order*
Chairman: Tommaso Lando

- Idir Arab: *Some properties of a class that contains increasing failure rate distributions*
- Beatriz Santos: *A new criterion for the star-shape order with applications*
- Tommaso Lando: *Increasing concave comparisons of order statistics with a nonparametric testing approach*

11:15-11:45 Coffee break

11:45-13:15 **Session 8** - *Estimation and DOE*
Chairman: Anna Dembinska

- Katherine Davies: *Stochastic EM Algorithm for Generalized Exponential Cure Rate Model and an Empirical Study*
- Omer Ozturk: *Order Restricted Randomization in Agricultural Field Experiments*
- Anna Dembińska: *Maximum likelihood estimators based on discrete component lifetimes of a k-out-of-n system*

13:15-15:00 Lunch

15:00-20:00 Guided tour to Amalfi coast

Friday 27^{th} **May**

8:30-9:30 **Plenary talk** (online)

Peng Zhao, Jiangsu Normal University, PR of China
Assessing Cyber Risks of PMU Network in a Smart Grid
Chairman: Franco Pellerey

9:30-11:00 **Session 9** - *Reliability and Aging*
Chairman: Jorge Navarro

- Marta Sánchez Sánchez: *A multivariate class of priors based on stochastic orders with application in reliability engineering*
- Magdalena Szymkowiak: *Sufficient and necessary conditions on system signatures assuring that the systems lifetimes preserve distributional properties of their components*
- Gema Pigueras: *Comparisons of relative spacings of order statistics in terms of the expected proportional shortfall order*

11:00-11:15 Coffee break

11:15-13:15 **Session 10** - *Record values and extreme values*
Chairman: Udo Kamps

- Tomasz Rychlik: *Bounds on moments of record values from iid continuous life distributions*
- Diana Rauwolf: *A record-values property associated with a renewal process*

- Amir Khorrami Chokami: *Extremal dependence between maxima of concomitants*
- Marie Kratz: *Multi-Normex approach based on ordered statistics for evaluating the sum of heavy tailed random vectors*

13:15-13:30 Closure Ceremony

13:30-15:00	Lunch

Contents

Some properties of a class that contains increasing failure rate distributions

I. Arab[a], P.E. Oliveira[a] and T. Lando[b,c]

[a] *CMUC, Department of Mathematics, University of Coimbra, Portugal.*

[b] *VSB Technical University of Ostrava, Czech Republic.*

[c] *Department of Economics, University of Bergamo, Italy.*

Abstract

We study a family of distributions characterized by the convexity of their odds function, or equivalently by the increasingness of their corresponding odds rate (IOR), showing that this is a necessary condition for the increasing hazard rate property (IHR) and coherent with the notion of "adverse ageing", in other words, the class of IOR distributions is larger than the one of IHR distributions. The IHR distributions are dominated by the unit exponential w.r.t the convex transform order (CTO) introduced in Van zwet, as for the IOR distributions, they are dominated by the log-logistic distribution, a distribution that is dominated by the unit exponential and possesses some nice properties with regard to its lack of memory. We prove some preservation properties of this class under several transformations that are often considered in reliability and life testing problems, including formation of order statistics. Moreover, the IOR assumption enables the derivation of survival bounds and tolerance limits (cf. Barlow 1964a, Barlow 1964b and Zimmer et al. 1998), extending the scope of applicability of some known results for IHR distributions .

References

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On Linear Prediction

N. Balakrishnan

McMaster University Hamilton, Canada

Abstract

In this talk, I will introduce the problem of optimal linear prediction. First, I will provide some historical results in this direction. Then, I shall describe some interesting (strange) properties of optimal predictors and their interpretations!

Confidence bands for exponential distribution functions under progressive type-II censoring

S. Bedbur

Institute of Statistics, RWTH Aachen University

Abstract

Based on a progressively type-II censored sample from the exponential distribution with unknown location and scale parameter, confidence bands for the underlying distribution function are constructed by using confidence regions for the parameters and Kolmogorov-Smirnov type statistics. Formulas for the boundaries and for the coverage probabilities of the confidence bands are derived, and the performance of the bands is compared in terms of band width and area by means of a data example. Extensions of the results to related models for ordered data, such as sequential order statistics, as well as to other underlying location-scale families of distributions are discussed.

References

1. Mies, F. & Bedbur, S. (2017) On the coverage probabilities of parametric confidence bands for continuous distribution and quantile functions constructed via confidence regions for a location-scale parameter. *Ann. Inst. Statist. Math.* **69(4)**, 925–944.

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Relaxing the increasing convex order: p_0-tail value at risk order

A.J. Bello Espina[a], J. Mulero[b], M.A. Sordo[a] and A. Suárez-Llorens[a]

[a] *Dpto. Estadística e Investigación Operativa, Universidad de Cádiz, 11510 Puerto Real, Spain*

[b] *Dpto. Matemáticas, Universidad de Alicante, Ap. 99, E-03080 Alicante, Spain*

Abstract

The tail value at risk at level p, with $p \in (0,1)$, is a risk measure that captures the tail risk of losses and asset return distributions beyond the p quantile. Given two distributions, it can be used to decide which is riskier. When the tail values at risk of both distributions agree, whenever the probability level $p \in (0,1)$, about which of them is riskier, then the distributions are ordered in terms of the increasing convex order. The price to pay for such unanimous agreement is that it is possible that two distributions cannot be compared despite our intuition that one is less riskier than the other. In this work, we introduce a family of stochastic orders, indexed by confidence levels $p_0 \in (0,1)$, that require agreement of tail values at risk only for levels $p > p_0$. We present its main properties and compares it with other families of stochastic orders that have been proposed in the literature to compare tail risks. We illustrate the results with a real data example.

Recent Advances on the Comparison of Dependent Random Variables

F. Belzunce[a], C. Martínez-Riquelme[a]

[a] *Universidad de Murcia, 30100 Espinardo (Murcia), Spain*

Abstract

In this talk we present some recent results on the topic of stochastic comparisons taking into account the possible dependence among the two random variables. We consider results in two directions. The first one considers comparisons of $X - Y$ and $Y - X$, being X and Y the two random variables to be compared. When we compare $X - Y$ and $Y - X$ in terms of the usual stochastic order we provide a non parametric test for this condition. Motivated by the application of this new test to real data sets we introduce a new joint stochastic order based on the comparison of $X - Y$ and $Y - X$ in terms of the increasing convex order. The other direction is the comparison of the residual lives $[X - t|X > t, Y > t]$ and $[Y - t|X > t, Y > t]$. Following the ideas of the weak joint hazard rate order, where the two previous random variables are compared in terms of the usual stochastic order, we consider a mean residual life type order by comparing the expected values $E[X - t|X > t, Y > t]$ and $E[Y - t|X > t, Y > t]$.

This work is supported by the Ministerio de Ciencia e Innovación de España bajo el proyecto PID2019-103971GB-I00/AEI/10.13039/501100011033.

A unified formulation of entropy and its application

N. Balakrishnan[a], F. Buono[b] and M. Longobardi[c]

[a] *McMaster University, Canada, bala@mcmaster.ca*
[b] *University of Naples Federico II, Italy, francesco.buono3@unina.it*
[c] *University of Naples Federico II, Italy, maria.longobardi@unina.it*

Abstract

A general formulation of entropy is proposed [1]. It depends on two parameters and includes Shannon, Tsallis and fractional entropy, all as special cases. This measure of information is referred to as fractional Tsallis entropy and some of its properties are then studied. In order to introduce the corresponding entropy in the context of Dempster-Shafer theory of evidence, the definition and some properties of the fractional Deng entropy are given [2]. Finally, it is presented an application of fractional version of Tsallis-Deng entropy to a classification problem.

Keywords. Measures of information, Shannon entropy, Tsallis entropy, Fractional entropy, Deng entropy, Dempster-Shafer theory of evidence.

References

1. Balakrishnan, N., Buono, F., Longobardi, M. (2022). A unfied formulation of entropy and its application. Accepted in *Physica A*.
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On the prediction of future failure times with quantile regression techniques

J. Navarro[a] and F. Buono[b]

[a] *Universidad de Murcia, Spain.*
[b] *University of Naples Federico II, Italy.*

Abstract

The prediction of the future failure times in a sample from early failures (type II censoring data) is a task of great interest [1,3]. A study on both the case of independent and dependent lifetimes is presented [2]. In both cases we assume identically distributed random variables. To predict the future failures, the quantile regression techniques are used and confidence regions for them are also provided. Some illustrative examples show how to apply the theoretical results to simulated and real data sets.

Keywords. Order statistics, Copula, Distorted distributions, Quantile regression, Type II censoring.

References

1. Bdair O.M., Raqab M.Z. (2022). Prediction of future censored lifetimes from mixture exponential distribution. *Metrika*. DOI: 10.1007/s00184-021-00852-z.

2. Navarro, J., Buono, F. (2022). Predicting future failure times by using quantile regression. *Submitted*.

3. Navarro J., Calì C., Longobardi M., Durante, F. (2022). Distortion representations of multivariate distributions. *Statistical Methods & Applications*. DOI: 10.1007/s10260-021-00613-2.

On the variability of some measures of uncertainty and discrimination

F. Buono [a], C. Calì [b], M. Longobardi [b]

[a] *Dipartimento di Matematica, Università di Napoli Federico II, Via Cintia, Complesso Monte S.Angelo, 80126 Naples, Italy*
[b] *Dipartimento di Biologia, Università di Napoli Federico II, Via Cintia, Complesso Monte S.Angelo, 80126 Naples, Italy*

Abstract

Kerridge inaccuracy measure [Kerridge 1961] and Kullback-Leibler divergence [Kullback, Leibler 1951] are useful tools to measure the discrepancy and the similarity between two random variables. In this presentation, in analogy with the concept of varentropy [Fradelizi *et al.* 2016], we introduce new indices of variability [Buono *et al.* 2021] for these measures. Some related properties, bounds and examples are presented. Finally, we show an application of Kullback-Leibler divergence and its dispersion index by using the mean-variance rule.

References

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3. Kerridge, D. F. (1961). Inaccuracy and inference. *J. R. Stat. Soc.* **23**, 184–194.

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Some remarks on the cumulative information generating function

A. Di Crescenzo[a], M. Capaldo[a] and A. Meoli[a]

[a] *Dipartimento di Matematica, Università degli Studi di Salerno, Via Giovanni Paolo II, 132, Fisciano (SA) 84084, Italy*

Abstract

An important concept in Information Theory is the entropy of a random variable, introduced by Shannon in 1948 aiming to describe the uncertainty in the outcome of a random experiment. Several generalizations and variations have been proposed in the literature. Among them, we recall the cumulative residual entropy introduced by Rao et al. [6] and the cumulative entropy by Di Crescenzo and Longobardi [3]. Recently, generalized and fractional versions of these information measures have been defined respectively in [5] and [7] for the cumulative residual entropy, and in [4] and [2] for the cumulative entropy. It is noteworthy the connections between these entropies and various notions of interest in reliability theory. Aiming to introduce a mathematical tool suitable to deal with these measures, we define the cumulative information generating function (CIGF). The latter allows to obtain some information measures related with the cumulative distribution function and the survival function. We study several results for the CIGF, with some bounds. We illustrate also some properties, including that the CIGF is a variability measure along the definition given in [1].

References

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Structural properties of (progressive) hybrid censoring schemes

E. Cramer

Institute of Statistics, RWTH Aachen, Germany

Abstract

Recently, Górny and Cramer (2018) have shown that (progressive) hybrid censoring schemes can be decomposed into some basic module types. In particular, this decomposition can be used to simplify the derivation of the distribution of the MLE for exponentially distributed lifetimes and leads to compact expressions of the MLE's density function in terms of B-splines.
A more detailed analysis of (progressive) hybrid censoring shows that further structural properties can be established which particularly explain similarities in both statistical inference and probabilistic analysis. In particular, the results obtained show that many findings can be directly deduced from the case of Type-I and Type-II censored data. Applications to likelihood inference, Bayesian estimation, and Fisher information are presented to illustrate impacts of these properties.

Stochastic EM Algorithm for Generalized Exponential Cure Rate Model and an Empirical Study

Katherine Davies[a], Joynob Siddiqua[b] and Suvra Pal[c]

[a] *University of Manitoba, Winnipeg, CANADA*
[b] *University of Manitoba, Winnipeg, CANADA*
[c] *University of Texas at Arlington, Texas, USA*

Abstract

In this talk, we consider two well-known parametric long-term survival models, namely, the Bernoulli cure rate model and the promotion time (or Poisson) cure rate model. Assuming the long-term survival probability to depend on a set of risk factors, the main contribution is in the development of the stochastic expectation maximization (SEM) algorithm to determine the maximum likelihood estimates of the model parameters. We carry out a detailed simulation study to demonstrate the performance of the proposed SEM algorithm through calculated bias, mean square error and coverage probability of the asymptotic confidence interval. For this purpose, we assume the lifetimes due to each competing cause to follow a two-parameter generalized exponential distribution, although one is free is choose any competing cause lifetime distribution. We also compare the results obtained from the SEM algorithm with those obtained from the well-known expectation maximization (EM) algorithm. For illustrative purposes, we analyze a breast cancer survival data and present the model fitting results for both estimation techniques studied here. Finally, we use a graphical method to assess the goodness-of-fit of the model with generalized exponential lifetimes.

Paradoxical aspects in probability, game theory and voting theory

Emilio De Santis[a] and Fabio Spizzichino[a]

[a] *Dipartimento di Matematica, Università di Roma La Sapienza, Italy*

Abstract

We address the questions of identifying 1-dependent Markov chains and some stopping times that realize an assigned majority graph. More precisely, for any digraph $G = ([n], \vec{E})$, that is, an oriented graph without self-loops and 2-cycles, one can construct a 1-dependent Markov chain and n identically distributed hitting times $\{T_1, \ldots, T_n\}$ on this chain such that the probability of the event $\{T_i > T_j\}$, for any $i, j = 1, \ldots, n$, is larger than $\frac{1}{2}$ if and only if $(i, j) \in \vec{E}$. We apply this result to answer to questions on a generalization of the Penney's game (see [1]).
In the second part of the talk we define the *ranking pattern* that can be seen as a multivariate generalizetion of the majority graph. We show that, for any given ranking pattern σ, one can explicitly construct a *load-sharing* dependence model that is concordant with σ. Such results can be applied in some different fields. In particular, we develop applications of them in the context of voting theory [2], [3], see also [4].

References

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Maximum likelihood estimators based on discrete component lifetimes of a k-out-of-n system

A. Dembińska[a] and K. Jasiński[b]

[a] *Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland, e-mail: dembinsk@mini.pw.edu.pl*

[b] *Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, Chopina 12/18, 87-100 Torun, Poland, e-mail: krzys@mat.umk.pl*

Abstract

So called k-out-of-n systems form an important class of systems studied in reliability theory. These systems consist of n elements and work as long as at least k of the elements function. Since such technical structures have some redundancy they find various applications in engineering when highly reliable products are needed.
In my talk I will consider maximum likelihood (ML) estimation of an unknown parameter of common distribution of component lifetimes of a k-out-of-n system. I will focus on the case when the component lifetimes are discrete and independent random variables. First I will present regularity conditions under which the ML estimators of interest exist almost surely for all sufficiently large n and are strongly consistent. Next I will look more closely at three typical discrete failure distributions—Poisson, binomial and negative binomial—and show that in these cases the ML estimators are unique, provided they exist, and that they are strongly consistent. Finally I will concentrate on the case when component lifetimes are geometrically distributed and will give a closed-form formula for the ML estimator in this case. Moreover, restricting the attention to n-out-of-n systems I will be able to derive some finite-sample properties of the ML estimator of the geometric parameter. In particular I will obtain its bias and mean squared error.
The talk will be based on results presented by Dembińska and Jasiński (2021, 2022).

References

1. Dembińska A. and Jasiński K. (2021) Maximum likelihood estimators based on discrete component lifetimes of a k-out-of-n system. *Test* **30**, 407–428.

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Some results on the weighted mean inactivity time function

Antonio Di Crescenzo[a], Abdolsaeed Toomaj[b]

[a] *Dipartimento di Matematica, Università di Salerno, Via Giovanni Paolo II n.132, I-84084 Fisciano (SA), Italy*
[b] *Faculty of Basic Sciences and Engineering, Department of Mathematics and Statistics, Gonbad Kavous University, Gonbad Kavous, Iran*

Abstract

The concept of mean inactivity time (MIT) plays a crucial role in reliability, risk theory and life testing. We recall that for a non-negative absolutely continuous random variable X denoting the lifetime of a system or a component, the MIT function of X is defined by

$$\widetilde{\mu}(t) = \mathbf{E}[t - X | X \le t] = \frac{1}{F(t)} \int_0^t F(x)\, \mathrm{d}x, \qquad t \in D,$$

where $D := \{t > 0 : F(t) > 0\}$ and $F(x) = \mathbf{P}(X \le x)$ is the cumulative distribution function of X. We introduce an extension of the MIT function, named *weighted mean inactivity time* (WMIT) function and defined as

$$\widetilde{\mu}_\psi(t) = \widetilde{\mu}_{\psi(X)}(t) = \mathbf{E}[\psi(t) - \psi(X) | X \le t] = \frac{1}{F(t)} \int_0^t \phi(x) F(x)\, \mathrm{d}x, \qquad t \in D,$$

where $\phi(x)$ is a non-negative and differentiable function for all $x \in [0, \infty)$, and where

$$\psi(x) := \int_0^x \phi(u)\, \mathrm{d}u, \qquad x \ge 0.$$

is the corresponding cumulative weight function.
We provide some connections among the variance of transformed random variables, the WMIT function and the weighted generalized cumulative entropy. The latter notion is a shift-dependent measure of uncertainty which is of interest in applied contexts. Moreover, we discuss some characterizations and preservation properties for stochastic comparisons based on the WMIT function.

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Prediction of Future Sports Records Based on Record Values

C. Empacher and U. Kamps

Institute of Statistics
RWTH Aachen University
52056 Aachen GERMANY

Abstract

The consideration and collection of data and its analysis is gaining more and more attention in several sports disciplines. For example, in US-sports like baseball, basketball and American football, data regarding many aspects of a game, team or player is recorded and analysed since many years. This trend can be observed in other areas of sports, such as athletics, as well. Predictions and forecasts are an interesting part in sports analytics. Previous works on forecasting athletic records are mainly based on extreme value theory; see, e.g., Einmahl and Magnus (2008), Noubary (2010), Einmahl and Smeets (2011), Fraga Alves et al. (2013), Stephenson and Tawn (2013). In this talk, maximum product of spacings prediction (Volovskiy and Kamps, 2020) is applied to predict future records based on data of previous ones. Moreover, exact and approximate prediction intervals (see, e.g., Raqab et al. (2007)) are shown and compared with regard to their expected lengths and their probabilities of coverage. The lower and upper record values are assumed to be based on a sequence of independent and identically distributed random variables having a power function distribution and a Pareto distribution, respectively. The choice of distributions is discussed. The results are applied to data from various disciplines of athletics as well as to American football data by considering the evaluation of players by assigning so-called fantasy football points to players' actions according to the points per reception scoring scheme (Empacher et al., 2022).

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Reversing Conditional Orderings

R. Foschi[a] and F. Spizzichino[b]

[a] *Università di Pisa, Dipartimento di Economia e Management*
[b] *Università degli Studi di Roma, La Sapienza*

Abstract

Stochastic orderings between random variables (or random vectors) constitute primary tools for the description and the characterization of concepts of stochastic dependence.
Specially in a statistical setting, the following problem is of interest: what can be said about dependence of X w.r.t. Y when we assume that Y is stochastically increasing w.r.t. X in some specified sense? Attention to this topic has been given several times in the literature under different stand-points or different languages.
Here we consider stochastic orderings between conditional distributions, also called *conditional orderings*. We will analyze some specific aspects concerning such orderings and relations among them.
For scalar random variables X and Y, we consider different conditional orderings of the form

$$\mathcal{L}(Y|X \in I) \leq_* \mathcal{L}(Y|X \in I'), \tag{1}$$

where I, I' are sets of different types and $\leq_*$ stands for $\leq_{st}$, $\leq_{hr}$ or $\leq_{lr}$. We analyze implications or equivalences concerning such relations.
A concept of *reversed conditional ordering* will in particular emerge from our discussion and our results will point out some symmetries existing between the mentioned univariate stochastic orderings $\leq_{st}$, $\leq_{hr}$, $\leq_{lr}$ and different types of conditional orderings (where a "type" of ordering can be defined in terms of the possible choices for the sets I, I' appearing in the above formula). More precisely, a *reversion* of a conditional ordering of the form (1) consists in one of the form

$$\mathcal{L}(X|Y \in J) \leq_{\widetilde{*}} \mathcal{L}(X|Y \in J'), \tag{2}$$

for suitable choice of the stochastic order $\leq_{\widetilde{*}}$ and of the sets J, J'.
It is known that conditional orderings define notions of positive dependence. We will analyze here positive dependence properties corresponding to the considered conditional orderings and we will see how results concerning implications and equivalences between conditional orderings can be translated in terms of dependence notions.

Then we will point out some direct applications of our results to dependence notions related with conditions of *default contagion* and to the case of *conditional independence* between X and Y. Suitable extensions to multivariate cases are possible.

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A general growth curve and its stochastic generalizations

G. Albano[a], A. Barrera[b], V. Giorno[c], P. Román-Román[d], F. Torres-Ruiz[d]

[a] *Dipartimento di Studi Politici e Sociali, Università degli Studi di Salerno, Via Giovanni Paolo II n. 132, I-84084 Fisciano (SA), Italy*
[b] *Departamento de Análisis Matemático, Estadística e Investigación Operativa y Matemática Aplicada, Facultad de Ciencias, Universidad de Málaga, Bulevar Louis Pasteur, 31, 29010 Málaga, Spain*
[c] *Dipartimento di Informatica, Università degli Studi di Salerno, Via Giovanni Paolo II n. 132, I-84084 Fisciano (SA), Italy*
[d] *Department of Statistics and Operations Research, Faculty of Sciences, University of Granada, 18071 Granada, Spain; Institute of Mathematics of the University of Granada (IMAG), Calle Ventanilla, 11, 18001, Granada, Spain*

Abstract

Stochastic models generalizing deterministic growth curves play an important role in modeling several dynamic phenomena, ranging from economics to medicine. The talk focuses on a quite general growth curve able to unify the classical cases of Malthusian, Richards, Gompertz, Logistic and some their generalizations. Here two stochastic models are obtained by introducing a moltiplicative and an additive noise to the deterministic equation. By considering a suitable parametrization of the growth equation, we show that the resulting processes are non homogeneous lognormal and gaussian respectively. For them we find the transition probability density function and the conditional and unconditional moments, so to prepare the field for the inference of the processes involved, as well as for the estimation of parameters.

Bounds on expected order statistics based on the monotone reversed failure rate distributions

Agnieszka Goroncy[a], Tomasz Rychlik[b]

[a] *Nicolaus Copernicus University*
Chopina 12/18, 87100 Toruń, Poland
[b] *Institute of Mathematics*
Polish Academy of Sciences
Śniadeckich 8, 00656 Warsaw, Poland

Abstract

Danielak [*Statistics* **27**, 305-324, (2003)], and Goroncy and Rychlik [*Metrika* **79**, 635–657, (2016)] presented the sharp positive upper mean-variance bounds on the expectations of order statistics with relatively high ranks based on independent identically distributed random variables with the decreasing and increasing failure rates, respectively. In this paper we determine analogous evaluations in the dual cases when the parent distributions have monotone reversed failure rates.

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Approximation of the empirical copula process revisited: A simple plug-in estimator

Oliver Grothe[a] and Fabian Kächele[a]

[a] *Karlsruhe Institute of Technology (KIT), Institute of Operations Research, Analytics & Statistics, Kaiserstraße 12, 76131 Karlsruhe, Germany.*

Abstract

Copulas are used in a wide domain of application and are usually estimated by the empirical copula. This estimator is simply derived from the ranks of data points in each dimension. Although the asymptotics of the empirical copula process are widely known, only few estimators for the covariance of this process are present. Therefore, we present a simple plug-in estimator for the covariance of the empirical copula process, i.e., the empirical copula itself. Additionally, an algorithm for the computation of the variance of arbitrary boxes in the domain of the empirical copula is presented. Thus, the variability of an estimate, e.g., of the survival copula or orthant probabilities can be determined. We illustrate the finite sample properties of the new estimator for dimensions $d \geq 2$ in a simulation study and give an outlook on possible applications.

Extremal dependence between maxima of concomitants

A. Khorrami Chokami[a] and M. Kratz[b]

[a] *University of Turin, Corso Unione Sovietica, 218 bis, 10134 Turin, Italy*
[b] *ESSEC Business School, CREAR Risk Research Center, Avenue Bernard Hirsch BP 50105, 95021 Cergy-Pontoise Cedex, France*

Abstract

The problem of finding methods to describe the extremal dependence among multiple time series has rapidly become attractive in recent years, due to the vast variety of fields where its practical implications are of interest. However, providing handy tools to assess such dependence is still challenging. An empirical method has been recently developed by Dacorogna and Cadena, where the authors provide a statistical approach to explore the dependence among extreme risks. The purpose is to develop further this problem from a theoretical point of view. The mathematical formalization that we propose involves the concept of concomitants of order statistics, widely studied in the literature. We focus on the asymptotic dependence between maxima of concomitants: Specifically, order a bivariate sequence of n i.i.d. random variables (X, Y) on the basis of the X-variable, and call the extreme set of the sequence (X_n, Y_n) the subset of couples where the first component is one of the k largest order statistics (k fixed). Consider the vector formed by the maxima of the concomitants belonging to the extreme set and to its complementary set. We study how the bivariate extremal dependence of (X, Y) influences the asymptotic joint distribution of the two maxima of concomitants. Revisiting a pivotal work by Joshi and Nagaraja, we propose an alternative way to tackle the problem, which allows us to consider the cases where upper tail dependence is present.

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Multi-Normex approach based on ordered statistics for evaluating the sum of heavy tailed random vectors.

M. Kratz[a] and E. Prokopenko[b]

[a] *ESSEC Business School, CREAR, Cergy-Pontoise 95021, France*
[b] *Sobolev Institute of Mathematics, Novosibirsk, Russia*

Abstract

Looking for the most accurate possible evaluation of the distribution of the sum of random variables, or vectors, or processes, with unknown distributions, has always been a classical problem in the probabilistic and statistical literature, with various answers depending on the given framework and on the specific application in view. On one hand, (uni- or multivariate) Central Limit Theorems (CLT) or Functional ones prove, under finite variance for the sum components or/and additional conditions, the asymptotic Gaussian behavior of the sum with some rate of convergence, focusing on the 'body' of the distribution. When considering heavy-tailed marginal distributions, Generalized CLT with the convergence to stable distributions, handle the case of infinite variance (see e.g. Samorodnitsky and Taqqu (1994) and Petrov (1995), and references therein), while, in the case of finite variance, an alternative way is to consider trimmed sums, removing extremes from the sample, to improve the rate of convergence; see e.g. Mori(1984) and Hahn(1991) and references therein.

When interested in tail distributions, CLTs may give poor results, especially when considering heavy tails. That is why different approaches have been developed, among which: (i) large deviation theorems, (ii) extreme value theorems (EVT) focusing on the tail only, (iii) hybrid distributions combining (asymptotic) distributions for both the main and extreme behaviors when considering independent random variables (see e.g. Csörgö et al. (1988), Zaliapin et al. (2005), Kratz (2014), and Müller (2019); we use the name given in Kratz (2014) for this type of hybrid distribution/method/approach, namely Normex distribution/method/approach.

Recall briefly the idea of Normex (for 'Norm(al)-Ex(tremes)') method. It consists of rewriting the sum of n random variables as the sum of their ordered statistics, and splitting it into two main parts, a trimmed sum removing the extremes, and the extremes. Using that the trimmed sum of the first $n-k-1$ ordered statistics is conditionnally independent of the k largest order statistics, given the $(n-k)$-th order statistics, we can express the distribution of the sum, integrating w.r.t. to the $(n-k)$-th order statistics and using a CLT for the conditional trimmed sum, and an EVT one for the k largest order statistics. Note that a benefit of Normex approach

is that it does not require any condition on the existence of moments, as the CLT applies on truncated random variables.
It is natural to extend the normex approach to a multivariate framework. With this goal of proposing a multi-normex method and distribution, we consider iid random vectors $\mathbf{X}_1, \ldots, \mathbf{X}_n$, with parent random vector $\mathbf{X}$ having a heavy-tailed d-dimensional distribution $F_{\mathbf{X}}$ and density $f_{\mathbf{X}}$ (when existing). Note that there are different ways to define multivariate extremes. The chosen way in this paper is w.r.t. the norm $\|\cdot\|$ in R^d, meaning that the ordered (w.r.t. the norm) vector of $(\mathbf{X}_1, \ldots, \mathbf{X}_n)$, denoted by $(\mathbf{X}_{(1)}, \ldots, \mathbf{X}_{(n)})$, satisfies

$$\|\mathbf{X}_{(1)}\| \leq \|\mathbf{X}_{(2)}\| \leq \cdots \leq \|\mathbf{X}_{(n)}\|.$$

We propose two versions of multi-normex. The first one, named *d-Normex*, is a natural extension to any dimension d of the univariate ($d = 1$) normex method as developed in Kratz (2014): We approximate the distribution of the trimmed sum via the CLT and consider the distribution of the maximum $\mathbf{X}_{(n)}$. This latter distribution is approximated via the Extreme Value theorem in the second multi-normex version, named *MRV-Normex*.

Aiming at proving the benefit of using a multi-normex distribution for a better fit of the whole (unknown) distribution F, assuming F heavy-tailed in the sense of $\|\mathbf{X}\| \in \mathcal{RV}_{-\alpha}$, *i.e.* regularly varying rv with $\alpha > 0$, we focus analytically on the case $\alpha \in (2; 3]$ (when $\|\mathbf{X}\|$ has a finite second moment, but no third moment), to compare the rates of convergence when using the CLT and the multi-normex approach, respectively.

Note our focus on heavy tailed distributions (i.e. distributions belonging to the max domain of attraction of Fréchet), where the impact of using Normex distribution will be much stronger than in the light tail case (because of the one big jump principle), in particular for risk analysis and management. We prove that the normex approach leads, as expected, to a better speed of convergence for evaluating the distribution of the sum than the CLT does, for such type of heavy-tailed distributions. When varying the fatness of the tail measured by $\alpha > 0$, we draw this comparison numerically, using e.g. geometrical multivariate quantiles.

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Increasing concave comparisons of order statistics with a nonparametric testing approach

T. Lando[a,b], I. Arab[c] and P.E. Oliveira[c]

[a] *University of Bergamo, Italy*
[b] *VŠB Technical University of Ostrava, Czech Republic*
[c] *University of Coimbra, Portugal*

Abstract

We deal with the problem of comparing the lifetimes of k-out-of-n systems with i.i.d. components with respect to ageing properties, by relying on the theory of stochastic orders. If we assume that the lifetime of each component is distributed according to a common parent distribution, say F, then the lifetime of the system is represented by the order statistic $X_{k:n}$, corresponding to a random sample of size n from F. Similarly to most decision problems in other research fields, the "best" performance of a k-out-of-n system may be represented by i) larger *magnitude*, or location, and ii) smaller risk or dispersion, since lifetime predictability is always preferable. Usually these aspects are considered separately, in terms of stochstic orders. Differently, the *increasing concave order* (ICV) enables comparisons of random variables in terms of both magnitude and dispersion, therefore combining aspects i) and ii) into a single preorder, that is, ICV expresses preference for the systems that have a tendency to have longer or, at least, less risky lifetimes. We determine sufficient conditions for ICV between order statistics, from the same or from different parent distributions, based on two related aspects, namely, i) ranks and sample sizes and; ii) information available on the parent. Conditions on i) consist of checking different inequalities (involving ranks and sample sizes), whereas those on ii) consist of checking whether the parent distribution belong to some classes, defined through the *convex transform order* of van Zwet (1964). Such classes include the *increasing failure rate* family and the *increasing odds rate* class. As a basic principle, larger classes (that is, less precise information on the parent) correpond to more stringent inqualities, and vice versa. Such an approach is suitable in a nonparametric framework, since there is no need to specify the model, once that we know that it belongs to one of these classes. Therefore, we propose nonparametric tests, to infer from data whether the parent distribution belongs to one of the classes considered, as this is a crucial aspect in our study. The suitable testing methodology may depend on the class, however, we mostly explore testing techniques that are based on the *greatest convex minorant*.

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Unbiased Probability Plots from Order Statistics with an Application to Extreme Value Distributions

A. Lepore[a]

[a] *Department of Industrial Engineering, Università degli studi di Napoli Federico II, Piazzale V. Tecchio, 80, 80125, Naples, Italy*
e-mail: antonio.lepore@unina.it

Abstract

Probability plots are still used in today's statistical software and analysis as a straightforward tool for graphical goodness-of-fit tests and outlier detection. In this work, probability plots for extreme value (EV) distributions are developed through the generalized least-squares estimation method and a convenient approximation of the first two moments of order statistics from the standard EV distributions. The proposed probability plots lead to unbiased graphical estimators of parameters and are compared with competitors by means of pivotal quantities that avoid the massive numerical investigations usually presented for similar purposes in the literature. Although more efficient biased solutions can be found theoretically, a Monte Carlo simulation study demonstrates the obtained parameter estimators achieve adequate performances in terms of mean square deviation from the true value. A real-case study regarding wind speed data collected at a candidate wind farm site in Southern Italy is also presented. The results demonstrate the proposed probability plot to effectively support EV analysis and assist practitioners in the selection of the optimal turbine class to be installed.

Asymptotic results for linear combinations of spacings generated by i.i.d. exponential random variables

C. Calì[a], M. Longobardi[a], C. Macci[b] and B. Pacchiarotti[b]

[a] *Dipartimento di Biologia, Università di Napoli Federico II, Via Cintia, Complesso Monte S. Angelo, 80126 Naples, Italy*

[b] *Dipartimento di Matematica, Università di Roma Tor Vergata, Via della Ricerca Scientifica, 00133 Rome, Italy*

Abstract

We prove large (and moderate) deviations for a class of linear combinations of spacings generated by i.i.d. exponentially distributed random variables (see e.g. [1] as a reference on large and moderate deviations). We allow a wide class of coefficients which can be expressed in terms of continuous functions defined on $[0, 1]$ which satisfy some suitable conditions. In this way we generalize some results proved in [3] which concern the empirical cumulative entropies defined in [2].

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Minimally stable lifetimes: marginal distributions of order statistics by means m.c.h.r functions and DD copulas

G. Nappo[a]

[a] *Sapienza University of Rome, Department of Mathematics "G.Castelnuovo", Piazzale Aldo Moro, 5, 00185, Rome, Italy*

Abstract

Let $T_1, ..., T_r$ be non-negative random variables (r.v.'s), and let $T_{1:r}$, $T_{2:r}, \cdots, T_{r:r}$ be the associated order statistics with marginal survival functions

$$\overline{G}_{k:r}(t) = \mathbf{P}(T_{k:r} > t), \quad t > 0,\ k \leq r.$$

As well known, the joint distribution of $T_1, ..., T_r$ can be described in terms of their survival copula K and of their marginal survival functions $\overline{G}_i(t) = \mathbf{P}(T_i > t)$. Furthermore, when $T_1, ..., T_r$ are absolutely continuous, their joint distribution can be described in terms of the associated multivariate conditional hazard rate (m.c.h.r.) functions. Therefore, conceptually, also the joint distribution, and the marginal distributions of the stochastic order statistics as well, could be expressed in terms either of K and $\overline{G}_i(t)$ or of the family of the m.c.h.r. functions.

In the first part of this talk we review a recent work [see Foschi, N., Spizzichino (2021)] concerning some detailed and simple formulas for $\overline{G}_{k:r}(t)$, in the case when $T_1, ..., T_r$ are *minimally stable*, i.e., for any subset $A = \{j_1, ..., j_\ell\} \subseteq \{1, 2, .., r\}$,

$$\mathbf{P}\big(\min_{j\in A} T_j > t\big) = \mathbf{P}\big(\min_{j\in\{1,...,\ell\}} T_j > t\big), \quad \forall\, t > 0.$$

The interest for the marginal distributions $\overline{G}_{k:r}$ within this class of r.v.'s has emerged in the field of systems' reliability, independently in Marichal, Mathonet, Waldhauser (2011), and in Navarro, Fernández-Sánchez (2020) [see also Navarro, Rychlik, Spizzichino (2021)].

Clearly *exchangeability* is a sufficient condition for minimal stability, whereas identical marginal distribution is a necessary one. When the common marginal distribution is continuous then minimal stability is equivalent to the requirement that the survival copula K be *diagonal dependent*, according to the definition in Navarro, Fernández-Sánchez (2020).

In the second part we discuss the following related questions

1) When the m.c.h.r. functions are given, how can one check whether the given model is minimally stable?

2) How can one construct a minimally stable model which is not exchangeable?

or equivalently

How can one construct a diagonal dependent copula which is not symmetric?

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Predicting record values by using bivariate distortions

J. Navarro[a,1]

[a] *Universidad de Murcia, Facultad de Matemáticas, Campus de Espinardo, 30100 Murcia, Spain. Email: jorgenav@um.es*

Abstract

1. Bivariate distortions.

The representation of univariate distributions as distortions of other univariate distributions has been a useful tool in the study of comparisons and aging properties of coherent systems (see [1]).
Similar representations can be obtained for bivariate distributions. The bivariate distortions were introduced in [3] as an alternative option to the classical copula representation. A bivariate distribution function $\mathbf{F}$ is a bivariate distortion of two univariate distribution functions G_1 and G_2 if it can be written as

$$\mathbf{F}(x, y) = D(G_1(x), G_2(y))$$

for all x, y, where $D : [0, 1]^2 \to [0, 1]$ is a bivariate distortion function, that is, it is 2-increasing, continuous and satisfies $D(0, v) = D(u, 0) = 0$ for all $u, v \in [0, 1]$ and $D(1, 1) = 1$. D can be extended to all the real numbers to be a bivariate distribution function with support included in $[0, 1]^2$. A similar representation holds for the respective reliability functions.
The above distortion representation is similar to a copula representation and so, many of their properties continue to hold. For example, it can be used to get the conditional distribution functions and to predict one variable from the other by using quantile regression techniques (see [3]).
However, note that D is not always a copula and that G_1 and G_2 are not always equal to the marginal distribution functions of $\mathbf{F}$. The main advantage is that these representations are more flexible and, sometimes, they allow us to get simple expressions. For example, in some cases, we can choose $G_1 = G_2$ even if the marginal distributions do not coincide.

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2. Predictions of record values.

We consider upper record values but similar results can be obtained for lower record values. If $X_1, X_2, \ldots,$ is a sequence of i.i.d. random variables with a common reliability (or survival) $\bar{F}$, then the first upper record is $R_1 = X_1$ and the ith record value for $i = 2, 3, \ldots$ is $R_i = X_j$ where j is the minimum value such that $X_j > R_{i-1}$. The goal is to predict R_s from R_r (or from $R_1, \ldots, R_r$) for $s > r \geq 1$.
To this end we will use a distortion representation for (R_r, R_s) by writing their joint reliability function $\bar{G}_{r,s}$ as

$$\bar{G}_{r,s}(x, y) = \widehat{D}_{r,s}(\bar{F}(x), \bar{F}(y))$$

for all $x \leq y$.
This representation will be used to get the conditional reliability function of $(R_s|R_r = x)$ and then to obtain quantile predictions for R_s and confidence intervals for that predictions. Specific results were obtained for the proportional hazard rate (PHR) model. In practice, the risk parameter of that model can be estimated by using the preceding record values. A case study in reliability is considered as well.
The results showed in this talk were obtained in the paper [2].

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Spacings in a step-stress model with censored exponential lifetimes

Maria Kateri[1,a], Nikolay I. Nikolov[1,b]

[1] *Institute of Statistics, RWTH Aachen University, Aachen, Germany 52056*

[a] *email: maria.kateri@rwth-aachen.de*

[b] *email: nikolay.nikolov@rwth-aachen.de*

Abstract

In step-stress accelerated life-testing (SSALT), the units of interest are tested under various stress conditions changed (usually increased) at different intermediate time points. It is commonly assumed that the lifetimes of the tested items are exponentially distributed on each stress level and the impact of the experienced stress change follows the cumulative exposure model. The main interest in a SSALT experiment lies mostly on inference for the mean lifetime (failure rate) at each stress level, [1]. Typically, these results are extrapolated to the normal operating conditions by using a life-stress relationship, for example the log-link connection, [2]. In this talk, the problem of testing the scale parameter in the SSALT model is considered by following the approach for constructing test procedures based on exponential spacings, proposed in [2]. For the simple SSALT under Type-II censoring, several modifications of the log-likelihood ratio statistic (LRS), derived in [3], are suggested by making use of the independent and scale invariant properties of the normalized spacings. The presented methods aim to eliminate the dependence of the exact LRS critical regions on the unknown lifetime parameters. The obtained results and LRS are further extended to the case of multilevel SSALT under the log-link assumption. With the help of a simulation study, it is shown that in some cases the presented test procedures have a power advantage under the multilevel experiment designs compared to the simple step-stress setup.

Keywords: Censored data; Likelihood-ratio test; Normalized exponential spacings; Scale invariant statistics.

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Comparisons of VaR and CoVaR in terms of the value of the conditional variable.

Patricia Ortega-Jimenez[a], Franco Pellerey[b], Miguel A. Sordo[a] and Alfonso Suárez-Llorens[a]

[a] *Department of Statistics and Operation Research, University of Cádiz, Spain. (Emails: patricia.ortega@uca.es, mangel.sordo@uca.es and alfonso.suarez@uca.es)*
[b] *Politecnico di Torino, 10129 Turin, Italy. Email: franco.pellerey@polito.it*

Abstract

Given a random vector (X, Y) and the risk levels $v \in [0, 1]$ and $u \in [0, 1]$, in order to study the risk of the component Y, two well known measures of risk are $VaR_v(Y)$ and $CoVaR_{v,u}(Y|X)$, that stands for $VaR_v(Y|X = VaR_u(X))$.

We compare the risk measures $VaR_v(Y)$ and $CoVaR_{v,u}(Y|X)$ in terms of the risk-level of the conditional variable, u. Sufficient conditions are provided, in terms of the copula, under which there exists $u^* \in (0, 1)$ such that, for all $u \geq u^*$, $CoVaR_{v,u}(Y|X)$ lies above $VaR_v(Y)$ for all $v \in (0, 1)$. The analytical expression of such u^* is provided for several bivariate copula families.

Order Restricted Randomization in Agricultural Field Experiments

Olena Kravchuk[a], Richard Jarrett[a], Omer Ozturk[b]

[a] *University of Adelaide, School of Agriculture, Food and Wine, Adelaide, Australia,*

[b] *The Ohio State University Department of Statistics 1958 Neil Avenue, Columbus OH, 43210, USA*

Abstract

Field experiments are run under two competing objectives, high precision, and minimal cost. The precision can be increased by either using sound experimentation techniques that account for the sources of variation with reasonable statistical models or increasing the sample size. Large sample sizes usually increase the cost of the experiment and may not be feasible. This paper uses order restricted randomized designs (ORRD) to increase the precision while keeping the sample size and cost of the experiment minimal. The ORRD described here starts with a randomized block design but adds a second layer of blocking by ranking plots within each block. This creates a two-way lay-out, blocks and ranking groups, and uses a restricted randomization to improve the precision of estimation of the treatment parameters. Ranking groups create a correlation structure for within-block units. The restricted randomization uses this correlation structure to reduce the error variance of the experiment. The paper computes the expected mean square for each source of variation in the ORRD design under a suitable model. It also provides approximate tests for treatment and ranking group effects. The efficiency of the ORRD is investigated through empirical power studies. Finally, an example based on a uniformity trial illustrates the use of the method in a split-plot experiment.

Keywords: Latin square, randomized block design, replicated Latin square, row-column design, order restricted randomization.

Comparisons of relative spacings of order statistics in terms of the expected proportional shortfall order

A. Castaño-Martínez[a], G. Pigueiras[b] and M.A. Sordo[c]

[a] [b] [c] *Department of Statistic and Operation Research, University of Cádiz, Spain*

Abstract

Relative spacings are relative differences between order statistics. Given two probability models, it is often of interest to study which one has larger relative spacings under different probabilistic criteria. In this work we obtain sufficient conditions in terms of the expected proportional shortfall order for the comparison of relative spacings. As an application, we compare income distributions in terms of relative deprivation and we characterize this condition within several parametric families.

Keywords: order statistics; expected proportional shortfall order; relative spacings; increasing convex order; relative deprivation

On the Evaluation of the Entropy of Fractionally Integrated Gauss-Markov Processes

M. Abundo[a], E. Pirozzi[b]

[a] *Dipartimento di Matematica, Università "Tor Vergata", via della Ricerca Scientifica, I-00133 Roma, Italy; abundo@mat.uniroma2.it*

[b] *Dipartimento di Matematica e Applicazioni, Università "Federico II", via Cintia, complesso Monte S. Angelo, I-80126 Napoli, Italy; enrica.pirozzi@unina.it*

Abstract

We focus on the estimation of the entropy of the dynamical system $\{X_\alpha(t), t \geq 0\}$, where the stochastic process $X_\alpha(t)$ is the *fractional Riemann-Liouville integral* of order $\alpha \in (0, 1)$ of a Gauss-Markov process. The numerical approximation of the entropy is obtained by means of an algorithm suitably devised in order to perform the simulation of sample paths of such processes and then by computing the approximation of the entropy called ApEn. Due to the main rule in the theory and applications played by the Brownian motion and the Ornstein-Uhlenbeck process, we firstly investigate the fractional integrals of such processes and their entropy. We put in evidence the relation between the value of α and the complexity degree; we show that the entropy of $X_\alpha(t)$ is a decreasing function of $\alpha \in (0, 1)$.
Furthermore, we aim to study the fractal dimension of the trajectories of $X^\alpha(t)$, in order to find its dependence from $\alpha \in (0, 1)$. We expect to obtain a similar behavior as for the entropy, namely this characteristic dimension should be a decreasing function of α.

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A record-values property associated with a renewal process

D. Rauwolf and U. Kamps

Institute of Statistics, RWTH Aachen University

Abstract

There is a well-known connection between order statistics and a common renewal process. Namely, the joint distribution of the first n occurrence times of a Poisson process conditioned on the event of n occurrences up to time t coincides with the joint distribution of n order statistics from a uniform distribution on the interval $(0, t)$ (see, e.g., Karlin and Taylor 1998, Resnick 1992). This so-called "order statistics property" and its extensions gave rise to a variety of articles studying related properties and characterizations of point processes (see, e.g., Debrabant 2012, Feigin 1979, Puri 1982, Shaked et al. 2004). In this talk, some respective results from the literature are exemplified and a record-values property associated with a renewal process is considered and analyzed. This fact leads to a class of joint distributions of ordered random variables containing the record model as a particular case. Several distributional examples along with relations to intermediate order statistics and a Liouville distribution are shown.

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Bounds on moments of record values from iid continuous life distributions

Tomasz Rychlik[a] and Magdalena Szymkowiak[b]

[a] *Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, 00656 Warsaw, Poland*

[b] *Institute of Automatic Control and Robotics, Poznań University of Technology, Plac Marii Skłodowskiej-Curie 5, 60965 Poznań, Poland*

Abstract

We consider the standard and kth record values arising in sequences of independent identically distributed continuous and positive random variables with finite expectations. We present the optimal upper bounds on the moments of various orders of nth values of kth records expressed in the scale units being the respective powers of the first population moment. The bounds depend on the type k of the record, its number n and the moment order α. We also present the conditions on the parameters k, n and α under which finiteness of the population mean does not guarantee finiteness of the αth moment of the nth value of kth moment.

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A new criterion for the star-shape order with applications

I. Arab[a], M. Hadjikyriakou[b], P.E. Oliveira[a] and B. Santos[a]

[a] *CMUC, Department of Mathematics, University of Coimbra, Portugal.*

[b] *University of Central Lancashire, Cyprus.*

Abstract

Given two random variables that represent the lifetime of two different systems, it is often desired to compare their ageing behaviours. The star-shape order, introduced by Barlow and Proschan (1975), captures the meaning of a system ageing faster than another. Thus, it becomes of interest to establish comparison criteria between two given distributions F and G, with respect to this order relation. Unfortunately, since the star-shape order depends on the star-shapedness of $G^{-1} \circ F$, this becomes rather difficult when the distributions or the quantile functions do not have explicit closed formulas or manageable ones. Saunders and Moran (1978) proposed a criterion to order families of distributions depending on one real parameter, with respect to this order relation. However, this criterion might not be applicable when considering the lifetime of complex systems, as these usually depend on several parameters. We obtain a new criterion for the star-shaped order by extending the result given by Saunders and Moran (1978) for families of distributions indexed by more than one parameter. This new criterion is then applied to obtain star-shape comparability between parallel and series systems with different component's behaviour, extending the results proved by Kochar and Xu (2011, 2014). These new results contrast with the ones obtain in Arab et al. (2020), where non-comparability w.r.t the convex transform order (stronger than the star-shape order) was established.

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A multivariate class of priors based on stochastic orders with application in reliability engineering

F. Ruggeri[a], M. Sánchez-Sánchez[b], M.A. Sordo[c] and A. Suárez-Llorens[d]

[a] *CNR Istituto di Matematica Applicata e Tecnologie Informatiche. Via Alfonso Corti, 12, 20133 Milano, Italy, fabrizio@mi.imati.cnr.it*

[b] *Facultad de Ciencias, Universidad de Granada. Av. de Fuente Nueva, s/n, 18071, Granada, Spain, martasanchez@ugr.es*

[c] *Facultad de Ciencias, Universidad de Cádiz. Avda. Rep. Saharaui s/n, 11510 Puerto Real, Cádiz, Spain, mangel.sordo@uca.es*

[d] *Facultad de Ciencias, Universidad de Cádiz. Avda. Rep. Saharaui s/n, 11510 Puerto Real, Cádiz, Spain, alfonso.suarez@uca.es*

Abstract

For the study of robust Bayesian analysis, we will introduce the uncertainty in multivarite prior distribution by using a new class of prior distribution, based on weighted distributions, stochastic orders and multivariate total positivity of order 2 (MTP2). Not only a new definition for the class will be provided but also its interpretation and the main properties. In addition, we will measure the uncertainty induced by that new class, as well as its effect on the posterior distribution, considering the Hellinger metric and the Kullback-Leibler divergence. In order to apply the new class, we will conclude with a real application about train door reliability in underground trains, showing the result obtained and an interpretation of these results.

Key Words: Robustness Bayesian Analysis, prior class, stochastic orders, weight functions, multivariate distributions.

Continuous-time statistics and generalized relaxation equations

E. Scalas[a]

[a] *Department of Mathematics, University of Sussex, Falmer, Brighton, UK*

Abstract

Using two simple examples, the continuous-time random walk as well as a two state Markov chain, the relation between generalized anomalous relaxation equations and semi-Markov processes is illustrated. This relation is then used to discuss continuous-time random statistics in a general setting, for statistics of convolution-type. Two examples are presented in some detail: the sum statistic and the maximum statistic.

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Comparing strong risk aversion in Yaari's dual theory of risk

A. Castaño-Martínez[a], C.D. Ramos[a], M.A. Sordo[a] and G. Pigueiras[a]

[a] *University of Cádiz, Facultad de Ciencias, Puerto Real, Cádiz*

Abstract

In Yaari's (1987) dual theory, attitudes towards risk are characterized by a function (called distortion) that modifies the underlying tail probabilities before calculating a generalized expected risk. In this context, Wang and Young (1998) provided motivation, both from the economic and statistical approaches, for a sequence of distortion-free partial orderings of risks. However, this approach may be inadequate to compare behaviors of strong risk aversion, where the agents are mainly concerned with losses above some threshold value. Our purpose is to overlap this gap and provide motivation, both from economic and statistical approaches, for weaker distortion-free partial orderings of risks when the agents behavior is focused on aversion to very large losses. The first non-trivial expected record value plays a central role in this framework.

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The method of multivariate conditional hazard rates: recent developments and applications.

Fabio Spizzichino

Sapienza University, Rome, Italy

Abstract

Attention will be essentially focused on some different aspects of stochastic dependence for an n-tuple of non-negative random variables $X_1, ..., X_n$. As usual, the symbols $X_{1:1}, ..., X_{n:n}$ will denote the corresponding order statistics. The variables $X_1, ..., X_n$ will be interpreted as *lifetimes* and their joint probability distribution can be generally described by means of the*joint survival function*$\overline{F}(s_1, ..., s_n) := P(X_1 > s_1, ..., X_n > s_n)$.
It will be also natural to assume that one-dimensional marginal distributions of $X_1, ..., X_n$ are continuous with invertible survival functions $\overline{G}_1, ..., \overline{G}_n$ respectively, so that stochastic dependence is normally described in terms of the corresponding *survival copula*

$$K(u_1, ..., u_n) := \overline{F}\left(\overline{G}_1^{-1}(u_1), ..., \overline{G}_n^{-1}(u_u)\right),$$

for $u_j \in [0, 1], j = 1, ..., n$. More specifically, we will focus on the absolutely continuos case, where the joint distribution is characterized by the joint density function $f(x_1, ..., x_n)$:

$$\overline{F}(s_1, ..., s_n) = \int_{s_1}^{+\infty} \int_{s_2}^{+\infty} .. \int_{s_n}^{+\infty} f(x_1, ..., x_n)\, dx_1 ... dx_n.$$

In such a case the no-tie condition $P(X_{1:1} < ... < X_{n:n}) = 1$ necessarily holds and, besides the survival copula K, one can also use the density function f as a tool to express properties of stochastic dependence. In particular it can be used to obtain the expressions for conditional probabilities of the form

$$P(X_{j_1} > s_{j_1}, ..., X_{j_k} > s_k | X_{i_1} = x_1, ..., X_{i_h} = x_h). \tag{3}$$

A central aspect of interest in this talk is the circumstance that -in the absolutely continuos case- the joint distribution of $X_1, ..., X_n$ can also be described in terms of the family of the *multivariate conditional hazard rate* (m.c.h.r.) functions, which are defined as follows:

$$\lambda_j(t|\emptyset) := \lim_{\Delta t \to 0^+} \frac{1}{\Delta t} P(X_j \leq t + \Delta t | X_{1:m} > t), \tag{4}$$

and, for $k = 1, ..., n-1$, $0 \leq t_1 < ... < t_k \leq t$,

$$\lambda_j(t|i_1, \ldots, i_k; t_1, \ldots, t_k) :=$$

$$\lim_{\Delta t \to 0^+} \frac{1}{\Delta t} P(X_j \leq t + \Delta t | X_{i_1} = t_1, \ldots, X_{i_k} = t_k, X_{k+1:m} > t.) \tag{5}$$

From an analytic viewpoint, the two descriptions are equivalent. In fact, in view of above definition, one can write a formula which ties the density to the m.c.h.r. functions and vice-versa.
However, such descriptions are really very different in nature, at least for respects that we can summarize as follows. Typically, for models whose density has a simple form, the m.c.h.r. functions can be rather involved, and vice-versa. In particular, models that emerge by imposing idealized conditions on the survival copula (e.g., Archimedean models) are very different from load-sharing models, which are described by most friendly m.c.h.r. functions.
One can claim furthermore that the two methods reveal to be respectively convenient to highlight different features of stochastic dependence. The method of the m.c.h.r. functions is specially apt to describe *dynamic* property of dependence. On the other hand it is not efficient, in general cases, to describe probabilistic objects such as marginal distributions, copulas, and conditional probabilities as in (3), when $\min(s_{j_1}, ..., s_k) < \max(x_1, ..., x_h)$.
Typically, both in applied probability and in the statistical literature, problems studied in terms of the m.c.h.r. functions are separated from those studied in terms of joint survival functions, joint density functions, copulas, conditional distributions and so on.

It is therefore interesting to single out conceptual problems, or applications, which could be analyzed by comparing and/or combining the two different methods.
A discussion of the above-mentioned aspects will be presented in the first part of the talk. It will be pointed out, in particular, that the m.c.h.r. functions can be useful for studying the following objects:

- the marginal probability distributions of minima of the form $M_{1:A} := \min_{j \in A} X_j$ and probabilities $\alpha_i(A)$ of events of type $(M_{1:A} = X_i)$, for $A \subseteq [n] \equiv \{1, 2, ..., n\}$, $i \in A$
- the probability distribution of the vector of the order statistics
- the probability distribution of the discrete random vector $\mathbf{J} \equiv (J_1, ..., J_n)$ where, for $i, r \in [n]$,
$$J_r = i \Leftrightarrow X_i = X_{r:m}.$$
We will also recall the definition of the afore-mentioned load-sharing models.

The second part of the talk will be devoted to an overview of some recent results and applications.
First we show a characterization of the condition of exchangeability in terms of the m.c.h.r. functions. This topic suggests some natural generalizations of exchangeability that still maintain relevant properties of symmetry. Discussion and examples will be based on the case of load-sharing models. Such models can be used also to establishing a comparison between the notion of exchangeability and the weaker notion of *Minimal Stability*. The latter is equivalent to identical marginal distributions for $X_1, ..., X_n$ combined with *Diagonal Dependence* of the survival copula. It is interesting the circumstance that such notion can be analyzed in terms of both the two different methods.
Other topics, to be surveyed as much as time permits, can be described as follows.
Concerning with the marginal survival functions $\overline{G}_{1:n}, ..., \overline{G}_{n:n}$ of the order statistics, in the specific case of Minimal Stability, we will consider different expressions respectively obtained in terms of the m.c.h.r. functions and of the *diagonal sections* of K. The comparison between the two expressions allows one to establish a bridge between the two methods and permits to combine them in the study of the order statistics.
The method of the m.c.h.r. functions and the special case of load-sharing models, also suggest introducing the concept of *reversed* multivariate conditional hazard rate (r.-m.c.h.r.) functions. Possible applications of r.-m.c.h.r. functions emerge in the analysis of the maximum among dependent variables and of *inactivity times* of coherent reliability systems.
Finally we aim to focusing on paradoxes related to the family of probabilities $\alpha_i(A)$. The connection with paradoxes of *voting theory* will be sketched and related applications of load-sharing models will be briefly illustrated.

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Characterization of distributions based on shape measures

Alfonso Suárez-Llorens

[a] *University of Cádiz, Facultad de Ciencias, Puerto Real, Cádiz*

Abstract

Three functional measures are introduced to examine the shape of univariate distributions based on the convex transform order. The first two ones are weighted tail indices and the third one is a skewness measure. We will present the main properties and show some examples. Also we will discuss the problem of estimation. Finally, we will show how these functional can be useful to characterize some classical distribution families as the generalized Pareto distribution or extreme value distributions.

Keywords: Convex transform order, kurtosis measures, shape measures, skewness measures, stochastic orders, tail weight, generalized Pareto distributions, extreme value distributions.

Some Information Measures for the Concomitants of m-GOS from a Gumbel's Bivariate Exponential Distribution

F. Suter[a,b], V. Preda[a,b,c] and I. Cernat[a]

[a] *University of Bucharest, Faculty of Mathematics and Computer Science, Academiei 14, 010014, Bucharest, Romania*

[c] *"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Calea 13 Septembrie 13, 050711 Bucharest, Romania*

[d] *National Institute of Economics Research "Costin C. Kiritescu", Calea 13 Septembrie 13, 050711 Bucharest, Romania*

Abstract

Generalized order statistics (GOS) is a concept introduced by Kamps [7] as a way of unifying various types of order statistics, and m-GOS is a special case of GOS, so that the marginal distributions can be obtained [7].
If we consider a sample from a bivariate distribution and an order statistics for the X's, the corresponding Y's are the concomitants of the order statistics [3]. Concomitants of FGM-family have been studied, including their information measures, see for example [2]. In this presentation we will take into consideration one of the Gumbel's bivariate exponential distributions [6] for which properties of the concomitants were derived in [1]. We will analyze the properties of these concomitants from the uncertainty point of view, i.e. obtaining some information measures such as classical Shannon [8] and Tsallis [10] entropies, but also their dynamic versions, residual [5] and past [4] entropies that have significance in the study of lifetime data. Therefore some applications of concomitants in Reliability are also discussed.

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Sufficient and necessary conditions on system signatures assuring that the systems lifetimes preserve distributional properties of their components

Barry C. Arnold[a], Tomasz Rychlik[b] and Magdalena Szymkowiak[c]

[a] *Department of Statistics, University of California, Riverside, USA*
[b] *Institute of Mathematics, Polish Academy of Sciences, Warsaw, Poland*
[c] *Institute of Automatic Control and Robotics, Poznan University of Technology, Poznań, Poland*

Abstract

We discuss distributional properties of a system with components whose lifetimes are identically distributed, and their joint distribution admits the Samaniego signature representation. We present the sufficient and necessary conditions on the system signatures under which the monotone failure rate and density of the components lifetimes are inherited by the system lifetime distribution.

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Optimal strategies for preventive maintenance of systems with dependent components of multiple types

N. Torrado[a]

[a] *Departamento de Análisis Económico: Economía Cuantitativa, Universidad Autónoma de Madrid, 28049 Madrid (Spain)*

Abstract

Preventive maintenance is one of the most popular maintenance strategies in reliability theory, whose purpose is to prevent system failure before it occurs. Most of the research works on optimal strategies for preventive maintenance have considered systems consisting of the same type of components. Recently, Hashemi et al. [2] investigated coherent systems with multiple types of independent components. In real situations, units never operate in isolation and can even share workloads, so it is important to take these dependencies into account. For the case of dependent components, Eryilmaz and Ozkut [1] investigated two optimization problems for parallel systems with multiple types of components. Specifically, they provided analytical expressions for two average cost rate functions, one for the optimal number of components and another for the optimal replacement time before system failure. In all these cases, the researchers compute numerically the optimal values for some specific cases, in order to optimize the corresponding objective functions. However, they do not provide any optimal solution valid for the general problem nor conditions that ensure their existence and uniqueness.
In this poster, we discuss some challenging open problems and conjectures recently proposed in [1] for parallel systems with dependent components of multiple types. Moreover, we present necessary conditions for the existence of the unique optimal value which minimizes the mean cost rate for two optimization problems.
The results presented in this poster have been recently accepted for publication in *Reliability Engineering and System Safety.*

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Assessing Cyber Risks of PMU Network in a Smart Grid

Peng Zhao[a]

[a] *Jiangsu Normal University, PR of China*

Abstract

The phasor measurement units (PMU) network has been utilized to monitor the system operation of a smart grid in recent decades. This brings a significant cyber risk for the power grid. In this talk, we discuss the risk levels of PMUs over the network which faces the cyber threats while considering the observation errors. The risk mitigation strategies for improving the safety levels of PMUs are also discussed.